# Package 'paramix'

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**Title** Aggregate and Disaggregate Continuous Parameters for Compartmental Models

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**Description** A convenient framework for aggregating and disaggregating continuously varying parameters (for example, case fatality ratio, with age) for proper parametrization of lower-resolution compartmental models (for example, with broad age categories) and subsequent upscaling of model outputs to high resolution (for example, as needed when calculating age-sensitive measures like years-life-lost).

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Create the Blending and Distilling Object

## **Description**

Based on model and output partitions, create a mixing partition and associated weights. That table of mixing values can be used to properly discretize a continuously varying (or otherwise high resolution) parameter to a relatively low resolution compartmental stratification, and then subsequently allocate the low-resolution model outcomes into the most likely high-resolution output partitions.

# Usage

```
alembic(
  f_param,
  f_pop,
  model_partition,
  output_partition,
  pars_interp_opts = interpolate_opts(fun = stats::splinefun, kind = "point", method =
        "natural"),
  pop_interp_opts = interpolate_opts(fun = stats::approxfun, kind = "integral", method =
        "constant", yleft = 0, yright = 0)
)
```

## Arguments

f\_param

a function, f(x) which transforms the feature (e.g. age), to yield the parameter values. Alternatively, a data.frame where the first column is the feature and the second is the parameter; see xy.coords() for details. If the latter, combined with pars\_interp\_opts to create a parameter function.

f\_pop

like f\_param, either a density function (though it does not have to integrate to 1 like a pdf) or a data.frame of values. If the latter, it is treated as a series of populations within intervals, and then interpolated with pop\_interp\_opts to create a density function.

model\_partition

a numeric vector of cut points, which define the partitioning that will be used in the model; must be length > 1

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```
output_partition
```

the partition of the underlying feature; must be length > 1

pars\_interp\_opts

a list, minimally with an element fun, corresponding to an interpolation function. Defaults to splinefun() "natural" interpolation.

pop\_interp\_opts

like pars\_interp\_opts, but for density. Defaults to approxfun() "constant" interpolation.

#### **Details**

The alembic function creates a mixing table, which governs the conversion between model and output partitions. The mixing table a data.table::data.table() where each row corresponds to a mixing partition  $c_i$ , which is the union of the model and output partitions - i.e. each unique boundary is included. Within each row, there is a weight and relpop entry, corresponding to

$$\mathrm{weight}_i = \int_{c_i} f(x) \rho(x) \mathrm{d}x$$

$$\mathsf{relpop}_i = \int_{c_i} \rho(x) \mathsf{d}x$$

where f(x) corresponds to the f\_param argument and  $\rho(x)$  corresponds to the f\_pop argument.

This mixing table is used in the blend() and distill() functions.

When blending, the appropriately weighted parameter for a model partition is the sum of weight<sub>i</sub> divided by the relpop<sub>i</sub> associated with mixing partition(s) in that model partition. This corresponds to the properly, population weighted average of that parameter over the partition.

When distilling, model outcomes associated with weighted parameter from partition j are distributed to the output partition i by the sum of weights in mixing partitions in both j and i divided by the total weight in j. This corresponds to proportional allocation according to Bayes rule: the outcome in the model partition was relatively more likely in the higher weight mixing partition.

# Value

a data.table with columns: model\_partition, output\_partition, weight and relpop. The first two columns identify partition lower bounds, for both the model and output, the other values are associated with; the combination of model\_partition and output\_partition forms a unique identifier, but individually they may appear multiple times. Generally, this object is only useful as an input to the blend() and distill() tools.

#### See Also

blend()
distill()

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# **Examples**

```
ifr_levin <- function(age_in_years) {
    (10^(-3.27 + 0.0524 * age_in_years))/100
}
age_limits <- c(seq(0, 69, by = 5), 70, 80, 101)
age_pyramid <- data.frame(
    from = 0:101, weight = ifelse(0:101 < 65, 1, .99^(0:101-64))
)
age_pyramid$weight[102] <- 0
# flat age distribution, then 1% annual deaths, no one lives past 101
ifr_alembic <- alembic(ifr_levin, age_pyramid, age_limits, 0:101)</pre>
```

blend

**Blend Parameters** 

## **Description**

blend extracts aggregate parameters from an alembic object.

# Usage

```
blend(alembic_dt)
```

# Arguments

```
alembic_dt an alembic() return value
```

### Value

a data.table of with two columns: model\_partition (partition lower bounds) and value (parameter values for those partitions)

```
ifr_levin <- function(age_in_years) {
    (10^(-3.27 + 0.0524 * age_in_years))/100
}

age_limits <- c(seq(0, 69, by = 5), 70, 80, 101)
age_pyramid <- data.frame(
    from = 0:101, weight = ifelse(0:101 < 65, 1, .99^(0:101-64))
)
age_pyramid$weight[102] <- 0
# flat age distribution, then 1% annual deaths, no one lives past 101
alembic_dt <- alembic(ifr_levin, age_pyramid, age_limits, 0:101)
ifr_blend <- blend(alembic_dt)</pre>
```

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```
# the actual function
plot(
  60:100, ifr_levin(60:100),
  xlab = "age (years)", ylab = "IFR", type = "l"
# the properly aggregated blocks
  age_limits, c(ifr_blend$value, tail(ifr_blend$value, 1)),
  type = "s", col = "dodgerblue"
# naively aggregated blocks
ifr_naive <- ifr_levin(head(age_limits, -1) + diff(age_limits)/2)</pre>
  age_limits, c(ifr_naive, tail(ifr_naive, 1)),
  type = "s", col = "firebrick"
# properly aggregated, but not accounting for age distribution
bad_alembic_dt <- alembic(</pre>
  ifr_levin,
  within(age_pyramid, weight <- c(rep(1, 101), 0)), age_limits, 0:101
ifr_unif <- blend(bad_alembic_dt)</pre>
lines(
  age_limits, c(ifr_unif$value, tail(ifr_unif$value, 1)),
  type = "s", col = "darkgreen"
)
```

distill

Distill Outcomes

# **Description**

distill takes a low-age resolution outcome, for example deaths, and proportionally distributes that outcome into a higher age resolution for use in subsequent analyses like years-life-lost style calculations.

## Usage

```
distill(alembic_dt, outcomes_dt, groupcol = names(outcomes_dt)[1])
```

# Arguments

```
alembic_dt an alembic() return value

outcomes_dt a long-format data.frame with a column either named from or model_from and a column value (other columns will be silently ignored)

groupcol a string, the name of the outcome model group column. The outcomes_dt[[groupcol]] column must match the model_partition lower bounds, as provided when constructing the alembic_dt with alembic().
```

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#### **Details**

When the value column is re-calculated, note that it will aggregate all rows with matching groupcol entries in outcomes\_dt. If you need to group by other features in your input data (e.g. if you need to distill outcomes across multiple simulation outputs or at multiple time points), that has to be done by external grouping then calling distill().

## Value

a data.frame, with output\_partition and recalculated value column

## **Examples**

```
ifr_levin <- function(age_in_years) {
    (10^(-3.27 + 0.0524 * age_in_years))/100
}

age_limits <- c(seq(0, 69, by = 5), 70, 80, 101)
age_pyramid <- data.frame(
    from = 0:101, weight = ifelse(0:101 < 65, 1, .99^(0:101-64))
)
age_pyramid$weight[102] <- 0
# flat age distribution, then 1% annual deaths, no one lives past 101

alembic_dt <- alembic(ifr_levin, age_pyramid, age_limits, 0:101)

results <- data.frame(model_partition = head(age_limits, -1))
results$value <- 10
distill(alembic_dt, results)</pre>
```

distill\_summary

Distillation Calculation Comparison Summary

# Description

Implements several approaches to imputing higher resolution outcomes, then tables them up for convenient plotting.

# Usage

```
distill_summary(alembic_dt, outcomes_dt, groupcol = names(outcomes_dt)[1])
```

# **Arguments**

```
alembic_dt an alembic() return value
```

outcomes\_dt a long-format data.frame with a column either named from or model\_from

and a column value (other columns will be silently ignored)

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groupcol

a string, the name of the outcome model group column. The outcomes\_dt[[groupcol]] column must match the model\_partition lower bounds, as provided when constructing the alembic\_dt with alembic().

#### Value

a data.table, columns:

- partition, the feature point corresponding to the value
- value, the translated outcomes\_dt\$value
- method, a factor with levels indicating how feature points are selected, and how value is weighted to those features:
  - f\_mid: features at the alembic\_dt outcome partitions, each with value corresponding to
    the total value of the corresponding model partition, divided by the number of outcome
    partitions in that model partition
  - f\_mean: the features at the model partition means
  - mean\_f: the features distributed according to the relative density in the outcome partitions
  - wm\_f: the alembic() approach

```
library(data.table)
f_param <- function(age_in_years) {
    (10^(-3.27 + 0.0524 * age_in_years))/100
}

model_partition <- c(0, 5, 20, 65, 101)
density_dt <- data.table(
    from = 0:101, weight = c(rep(1, 66), exp(-0.075 * 1:35), 0)
)
alembic_dt <- alembic(
    f_param, density_dt, model_partition, seq(0, 101, by = 1L)
)

# for simplicity, assume a uniform force-of-infection across ages =>
# infections proportion to population density.
model_outcomes_dt <- density_dt[from != max(from), .(value = sum(f_param(from) * weight)),
    by = .(model_from = model_partition[findInterval(from, model_partition)])
]

ds_dt <- distill_summary(alembic_dt, model_outcomes_dt)</pre>
```

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Interpolation Options

# **Description**

Creates and interpolation options object for use with alembic().

# Usage

```
interpolate_opts(fun, kind = c("point", "integral"), ...)
```

# Arguments

fun	a function
kind	a string; either "point" or "integral". How to interpret the x, y values being interpolated. Either as point observations of a function OR as the integral of the function over the interval.
	arbitrary other arguments, but checked against signature of fun

#### **Details**

This method creates the interpolation object for use with alembic(); this is a convenience method, which does basic validation on arguments and ensures the information used in alembic() to do interpolation is available.

The ... arguments will be provided to fun when it is invoked to interpolate the tabular "functional" form of arguments to alembic(). If fun has an argument kind, that parameter will also be passed when invoking the function; if not, then the input data will be transformed to  $\{x,z\}$  pairs, such that  $x_{i+1}-x_i*z_i=y_i$  - i.e., transforming to a point value and a functional form which is assumed constant until the next partition.

#### Value

a list, with fun and kind keys, as well as whatever other valid keys appear in . . . .

```
interpolate_opts(
   fun = stats::splinefun, method = "natural", kind = "point"
)
interpolate_opts(
   fun = stats::approxfun, method = "constant", yleft = 0, yright = 0, kind = "integral"
)
```

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parameter\_summary

Parameter Calculation Comparison Summary

## **Description**

Implements several approaches to computing partition-aggregated parameters, then tables them up for convenient plotting.

## Usage

```
parameter_summary(f_param, f_pop, model_partition, resolution = 101L)
```

# **Arguments**

f_param	a function, f(x) which transforms the feature (e.g. age), to yield the parameter
---------	--

values. Alternatively, a data.frame where the first column is the feature and the second is the parameter; see xy.coords() for details. If the latter, combined with none interest and the second seco

with pars\_interp\_opts to create a parameter function.

f\_pop like f\_param, either a density function (though it does not have to integrate to

1 like a pdf) or a data.frame of values. If the latter, it is treated as a series of populations within intervals, and then interpolated with pop\_interp\_opts to

create a density function.

model\_partition

a numeric vector of cut points, which define the partitioning that will be used in

the model; must be length > 1

resolution the number of points to calculate for the underlying f\_param function. The

default 101 points means 100 partitions.

# Value

a data.table, columns:

- model\_category, a integer corresponding to which of the intervals of model\_partition the x value is in
- x, a numeric series from the first to last elements of model\_partition with length resolution
- method, a factor with levels:
  - f\_val: f\_param(x)
  - f\_mid: f\_param(x\_mid), where x\_mid is the midpoint x of the model\_category
  - f\_mean: f\_param(weighted.mean(x, w)), where w defined by densities and model\_category
  - mean\_f: weighted.mean(f\_param(x), w), same as previous
  - wm\_f: the result as if having used paramix::blend(); this should be very similar to mean\_f, though will be slightly different since blend uses integrate()

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```
# COVID IFR from Levin et al 2020 https://doi.org/10.1007/s10654-020-00698-1
f_param <- function(age_in_years) {</pre>
  (10^{-3.27} + 0.0524 * age_in_years))/100
densities <- data.frame(</pre>
  from = 0:101,
  weight = c(rep(1, 66), exp(-0.075 * 1:35), 0)
model_partition <- c(0, 5, 20, 65, 101)
ps_dt <- parameter_summary(f_param, densities, model_partition)</pre>
ps_dt
ggplot(ps_dt) + aes(x, y = value, color = method) +
  geom_line(data = function(dt) subset(dt, method == "f_val")) +
  geom_step(data = function(dt) subset(dt, method != "f_val")) +
  theme_bw() + theme(
   legend.position = "inside", legend.position.inside = c(0.05, 0.95),
   legend.justification = c(0, 1)
  ) + scale_color_discrete(
    "Method", labels = c(
      f_{val} = "f(x)", f_{mid} = "f(mid(x))", f_{mean} = "f(E[x])",
      mean_f = "discrete E[f(x)]", wm_f = "integrated E[f(x)]"
   )
  ) +
  scale_x_continuous("Age", breaks = seq(0, 100, by = 10)) +
  scale_y = 10^{10}("IFR", breaks = 10^c(-6, -4, -2, 0), limits = 10^c(-6, 0))
```

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